

coordinate; τ , shear stress; $F_{\ell}(\tau)$, integral moments of rheological function $f(\tau)$; τ_w , stress at horizontal substrate; τ_v , stress at vertical wall; x_0, v_0 , coordinate and velocity of motion of film edge; $Q(t)$, liquid influx; α, β , self-similarity exponents; $\varphi(\xi)$, dimensionless function of dimensionless argument; $B(a, b)$, beta function; C , numerical constant; d , distance between vertical planes.

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SPREADING OF SMALL LIQUID DROPS ALONG A FLAT SURFACE

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We discuss two limiting spreading laws for small drops of a viscous liquid which are supported by the experimental data.

Drops of coolant are sprayed onto a surface in order to obtain "soft" cooling. Calculation of the effectiveness of the heat transfer in this case is not possible without a knowledge of the processes of wetting and spreading of the drops adhering to the surface. Because of their small size, one can assume that the drops are isothermal with a variable temperature.

A phenomenological method of describing the spreading of a partially wetting liquid in the viscous regime was given in [1]. In this method an additional body force (equal to the gradient of the chemical potential) is introduced into the equations of hydrodynamics. For an incompressible liquid under isothermal conditions [2] this approach is equivalent to introducing an additional "disjoining" pressure [3, 4].

The equations of motion of a viscous liquid, written in the approximation of the theory of lubrication [5], together with the boundary conditions on the surface of the liquid drop, can be solved for the function $h(x, y, t)$ determining the shape of the free surface [6] [x and y are Cartesian coordinates in the plane of the solid surface $z = 0$; t is the time; the shape of the free surface of the liquid is given by the equation $z = h(x, y, t)$].

We assume that the shape of the drop is axially symmetric. With no loss of generality, we can take the line $x = y = 0$ as the symmetry axis. Let $a(t)$ be the radius of the circle wetted by the liquid on the solid surface $z = 0$. Then the equation of the line of three-phase contact is $x^2 + y^2 = a^2$. For further analysis it will be convenient to introduce polar coordinates: $r^2 = x^2 + y^2$, $\varphi = \arctg(x/y)$. By symmetry $h = h(r, t)$. If we then introduce the "local" coordinate $x^* = a(t) - r$, we find that close to the surface of the drop ($x^* \rightarrow +0$) the equation for the shape of the free surface reduces to the corresponding equation for the "plane" case considered in [1], which can be written (in dimensionless variables)

$$\frac{1}{2} \frac{\partial^2 \beta}{\partial \eta^2} + R \frac{\partial}{\partial \eta} [\Phi_0(\eta, \beta)] + \frac{u}{\alpha \eta^2} = 0, \quad u \equiv \frac{da^*}{dt^*}. \quad (1)$$

Here $L = \sqrt{\sigma/\rho g}$ is taken as the unit of length; $T = 3 \mu L/\sigma$ is the unit of time; $\beta = \alpha^2 - \theta^2 \geq 0$, $\alpha(\eta, t) \geq 0$ is the angle of inclination of the surface of the drop to the solid surface (in the approximation considered here $\alpha^2 \approx (\partial h/\partial x^*)^2$); θ is the equilibrium value of the

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wetting angle; R is a dimensionless constant characterizing the magnitude of the chemical potential (according to the estimates of [1] $R \sim 10^{-12}$); $\eta = h/L$; $\alpha^* = \alpha/L$; $t^* = t/T$. The function $\Phi_0(\eta, \beta)$ determines the dependence of the chemical potential of the liquid particles on the surface of the liquid film near the line of three-phase contact. Following [7, 8], it was assumed in [1] that $\Phi_0 = \eta^{-3}\beta$. The detailed analysis of (1) provided in [1] shows that the asymptotic model of the spreading of partially wetting liquids is complete and consistent. It was found, for example, that for $\eta = 0$ (i.e., on the line of three-phase contact) the Young condition $\beta = 0$ is always satisfied.

Putting $\beta \rightarrow 0$, $\alpha \approx 0$ in (1) and redefining the dependent variable $\beta = (u/\theta)\beta^*$, we can eliminate the quantities u and θ from (1). Hence in the limit $\alpha \rightarrow 0$ we have the direct proportionality $\beta \sim u/\theta$. As shown in [1], the condition $\alpha \approx \theta$ is satisfied everywhere where (1) holds if $u/\theta^3 \ll 1$.

In the other limiting case $\beta \gg \theta^2$, $\alpha^2 \gg \theta^2$ we can put $\alpha \approx \beta^{1/2}$. It then follows from (1) that $\beta \sim u^2/3$.

An approximate solution of (1), valid for arbitrary values of the variable $\eta \geq 0$ and the spreading rate $u \geq 0$, is found by using the weak logarithmic monotonically increasing nature of the function $\alpha(\eta)$ for $u > 0$, as established in [1, 9]. Hence we can put $\alpha \approx \text{const}$ in the last term in (1). With the above expression for $\Phi_0(\eta, \beta)$, (1) becomes linear. The solution in this case has been studied in [1] and for $1 \gg \eta \gg \sqrt{R} \sim 10^{-6}$ it can be written as

$$\alpha\beta = -u(\gamma + \ln R/\eta^2) + O(\eta), \quad (2)$$

where γ is Euler's constant. Following modern convention [4, 10], we define the dynamical wetting angle as the angle of inclination of the surface of the drop at a certain fixed height $\eta = \eta_0$, $\sqrt{R} \ll \eta_0 \ll 1$ (i.e., at a height which is macroscopically small, but yet so large that the disjoining pressure is negligibly small at $\eta = \eta_0$). Then, neglecting quantities of order $O(\sqrt{R})$, we obtain

$$\alpha_d \beta_d = u\omega, \quad \omega = -\gamma - \ln R/\eta_0^2 > 0, \quad (3)$$

where $\alpha_d = \alpha(\eta_0)$, $\beta_d = \beta(\eta_0)$.

Following [11], we assume that the shape of small drops is nearly spherical. In this case

$$a^3 = \frac{3V}{\pi} \frac{(1 + \cos \alpha_0)^2}{\sin \alpha_0 (2 + \cos \alpha_0)}. \quad (4)$$

Here α_0 is the dynamical wetting angle and V is the volume of the drop. Below we will identify $\alpha_0 = \alpha_d$. In the approximation of (1), relation (4) can be rewritten as $a^3 = 4V/\pi\alpha_d$. Differentiating this last relation with respect to time, we obtain

$$\frac{da_d}{d\tau} = -\left(\frac{3}{2}\right)^{1/3} \alpha_d^{4/3} u. \quad (5)$$

Here $\tau = \sigma t/2\mu\alpha_0$, $a_0 = (3V/4\pi)^{1/3}$.

Substituting (3) into (5), we can transform (5) to the form

$$\frac{d\alpha_d}{d\tau} = -\left(\frac{3}{2}\right)^{1/3} \alpha_d^{13/3} (1 - \theta^2/\alpha_d^2) \omega^{-1}.$$

Integrating this relation, we have

$$\tau = \left(\frac{2}{3}\right)^{1/3} \frac{3}{10} \omega \alpha_d^{-10/3} \chi(\theta^2/\alpha_d^2), \quad (6)$$

where the function $\chi(\theta^2/\alpha_d^2)$ is given as the series $\chi = \sum_{i=0}^{\infty} (\theta^2/\alpha_d^2)^i (1+0,6i)^{-1}$. When $\theta^2/\alpha_d^2 \ll 1$, $\chi \approx 1$, and when $\theta^2/\alpha_d^2 \rightarrow 1$ the function χ has a logarithmic singularity.

As noted, the constant ω can be determined for the dimensionless height η_0 at which the dynamical wetting angle α_d is defined. At this height the interior and exterior asymptotic

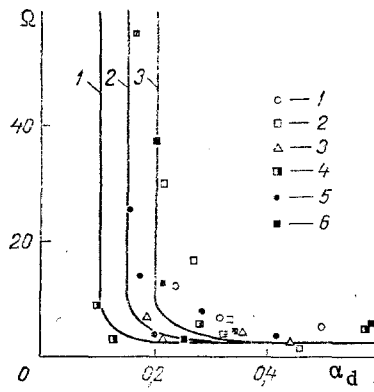


Fig. 1. Quantity $\Omega = \tau \alpha_d^{10/3}$ as a function of dynamical wetting angle α_d for the spreading of glass drops with different admixtures of TiO_2 and for different temperatures: 1, 2, 3) $T = 1010$ K; 5, 10, 20% TiO_2 , respectively; 4) $T = 1105$ K; 10% TiO_2 ; 5, 6) $T = 1180$ K and 5, 10% TiO_2 ; the theoretical curves 1, 2, 3 correspond to $\theta = 0.1$; 0.15; 0.2.

solutions of the spreading problem are joined together. Putting $\eta_0 = 10\sqrt{R}$, for example, we obtain $\omega \approx 4$ from (3). However, in comparing the theoretical relations with the experimental data, one must keep in mind that the quantities η_0 and R are somewhat uncertain because of the fact that the height at which the contact angle is determined experimentally is not known exactly and also because of the nearly complete lack of information on the magnitude of molecular interactions in particular cases for partial wetting. Nevertheless, because of the weak logarithmic dependence of ω [given by (3)], this quantity can be estimated fairly accurately by obtaining the best fit to the experimental data. It then follows from (3) that ω is of order one.

Experimental data was presented in [11] on the dependence of the dynamical wetting angle on spreading time of glass drops with TiO_2 admixtures for different temperatures on a specially prepared platinum plate (Figs. 2 and 3 of [11]). Figure 1 shows the dependence of the quantity $\Omega = \tau \alpha_d^{10/3}$ on the angle α_d from the experimental data and relation (6), where it was assumed that $0.3(2/3)^{1/3}\omega = 2.5$. The agreement between the experimental data and the theoretical calculations can be considered as completely satisfactory when one takes into account the complexity of determining the equilibrium contact angle θ from dynamical experiments of the type carried out in [11] (compare [12]) and also the relative uncertainty in ω .

Study of the function $\chi(\theta^2/\alpha_d^2)$ and the experimental data (Fig. 1) show that the spreading process is divided into two stages. In the first stage $\alpha_d^2 \gg \theta^2$. In this case $\chi = 1$ and the quantity $\Omega = \text{const}$ and does not depend on the equilibrium angle θ . Therefore, $a = \xi \omega^{-0.1} a_0^{0.9} (\sigma t / \mu)^{0.1}$, $\xi \approx 1.86$. Since ω is of order unity, we can write $a \approx 2a_0^{0.9} (\sigma t / \mu)^{0.1}$, i.e., the 1/10 law holds which is widely supported experimentally [13, 14]. This first stage of spreading of liquid drops can be called the power-law stage, since the dependence $a \sim t^{0.1}$ has the form of a power law.

In the second stage $\alpha_d^2 \rightarrow \theta^2$. In this case (6) is rewritten in the form $\tau = 2(2/3)^{1/3} \alpha_d^{-10/3} \omega \ln(\alpha_d - \theta)^{-1}$. Therefore, in this stage of the spreading we have $a = (16/3)^{1/3} a_0 [\theta \cdot (1 + \exp(-\kappa \sigma t / \mu))^{-1/3}]$, $\kappa = \theta^{10/3} (2/3)^{1/3} (4\omega a_0)^{-1}$, and hence this stage is called the exponential stage.

Finally, we note that the power-law stage is the more important in practice, since in the exponential stage the boundary of the drop is practically motionless. The exponential stage of spreading can become significant for small $\theta \ll 1$.

NOTATION

σ , surface tension; ρ , density; g , acceleration of gravity; μ , dynamical viscosity.

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DECREASE OF RESISTANCE IN A MICROPOLAR LIQUID

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Modified equations of motion of a micropolar liquid are derived. It is shown that, in nonlinear problems, a lower resistance is possible in comparison with an ordinary liquid, even in the case of laminar flow.

There are many ways to describe rheologically complex liquids, where the asymmetry of the stress tensor and the deformation rate tensor (viscoelastic liquids), the relaxation terms (Maxwellian liquid), etc. are accounted for to a greater or lesser degree. The phenomenological derivation of the equations [1] where only the asymmetry of the stress tensor is considered already leads to an extremely complex system of 19 partial differential equations with 22 viscosity coefficients. Introduction of the additional assumption of isotropy made it possible to reduce this system to seven equations [2, 3], which are at present widely used for describing liquids with polymer additions, liquid crystals, blood, etc. (see, for instance, surveys [4, 5]).

Asymmetric hydromechanics [2, 3] is characterized by the nonsymmetric stress tensor σ_{ij} and the additional tensor of micromoments m_{ij} :

$$\sigma_{ij} = -p\delta_{ij} + \mu(\partial_i V_j + \partial_j V_i) + k(\partial_i V_j - \epsilon_{ijm}\Omega_m), \quad (1)$$

$$m_{ij} = \alpha\delta_{ij} \operatorname{div} \Omega + \beta\partial_j \Omega_i + \gamma\partial_i \Omega_j, \quad \partial_i = \partial/\partial x_i. \quad (2)$$

Thus, besides the coefficient of dynamic viscosity μ , there are in asymmetric hydrodynamics three additional rotational viscosity coefficients and the coefficient k , which provides the measure of a particle's "coupling" with its ambient. The dilatational viscosity coefficient does not figure in (1), since we limit our considerations here to the case of incompressible liquids. It is evident from (1) and (2) that, in Eulerian presentation, a state in asymmetric hydromechanics is determined not only by the field of velocities V , but also by the field of angular velocities of microrotation Ω .

The equations of motion in asymmetric hydromechanics are given by

$$\rho \frac{dV}{dt} = (\mu + k)\Delta V + k \operatorname{rot} \Omega - \nabla p, \quad \operatorname{div} V = 0, \quad (3)$$

$$\rho J \frac{d\Omega}{dt} = (\alpha + \beta) \operatorname{grad}(\operatorname{div} \Omega) + \gamma \Delta \Omega - 2k\Omega + k \operatorname{rot} V. \quad (4)$$

The microinertial characteristics of the medium were not taken into account in [2] [the left-hand side of Eq. (4) was assumed to be zero], while Eq. (4) was used in [3] and the subsequently published papers (see the literature cited in [4, 5]).